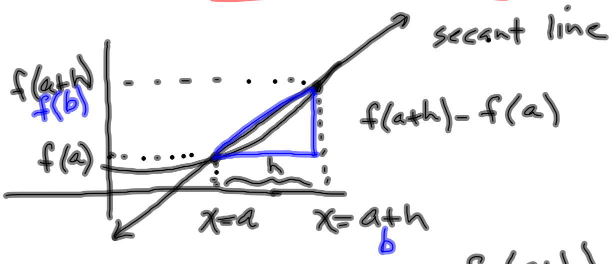
Review 6 Formal Definition of Derivative



slope of secant $= \dfrac{f(a+h) - f(a)}{h}$

slope of tangent

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a} = f'(a)$$

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ex. $\displaystyle \lim_{h \to 0} \frac{\ln(2+h) - \ln(2)}{h} = f'(2)$

equals $\frac{1}{2}$

where

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2}$

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Ex 2. Let $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

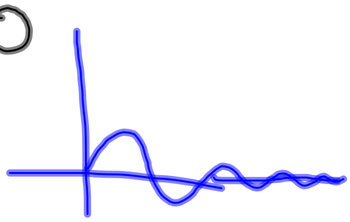
a) use the formal definition of the derivative to show that $f(x)$ is differentiable at $x=0$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{f(h) - 0}{h}$$

0 • oscillates

$$= \lim_{h\to 0} \frac{h^{\cancel{2}} \sin\frac{1}{h}}{\cancel{h}} \qquad \frac{1}{h} \to \infty$$

$$= 0$$



$$f'(0) = 0$$

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b) find $f'(x)$, $x \neq 0$

$f(x) = x^2 \sin(\frac{1}{x})$ $x \neq 0$

$f'(x) = x^2 \cdot \left(-\frac{1}{x^2}\right) \cdot \cos\left(\frac{1}{x}\right) + \sin\frac{1}{x} \cdot 2x$

$$f'(x) = \begin{cases} -\cos\frac{1}{x} + 2x\sin\frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

c) show $f'(x)$ is <u>not</u> continuous at $x=0$

1. $\lim_{x\to 0} f'(x) = \not\exists$ by oscillation

2.

3.

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